Paper Reference(s)

6680/01

Edexcel GCE

Mechanics M4

Advanced Level

Wednesday 22 June 2011 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

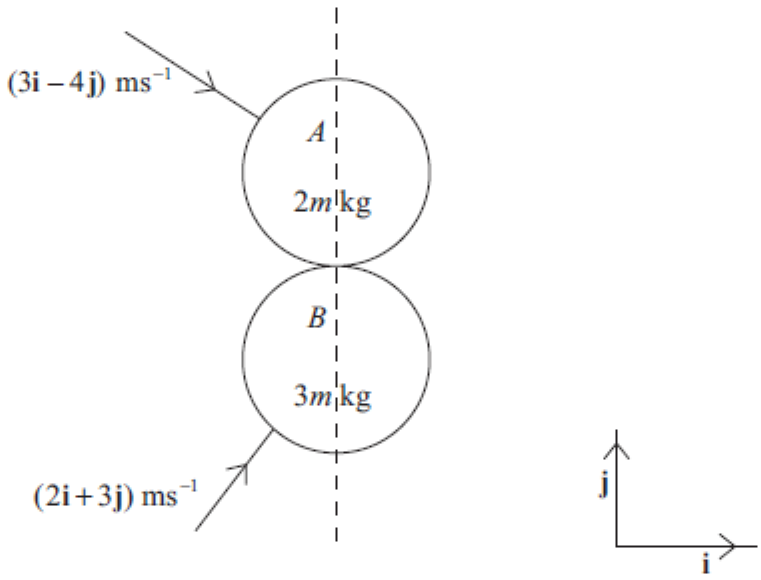

Figure 1

Two smooth uniform spheres A and B have masses $2m$ kg and $3m$ kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1} and sphere B has velocity $(2\mathbf{i} + 3\mathbf{j})$ m s^{-1}. When the spheres collide, the line joining their centres is parallel to $\mathbf{j}$, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{3}{7}$.

Find, in terms of m, the total kinetic energy lost in the collision.

(10)

2.

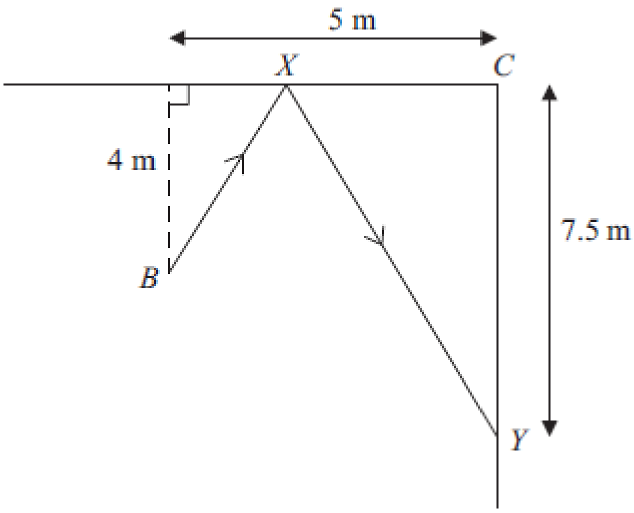

Figure 1

Figure 2 represents part of the smooth rectangular floor of a sports hall. A ball is at *B*, 4 m from one wall of the hall and 5 m from an adjacent wall. These two walls are smooth and meet at the corner *C*. The ball is kicked so that it travels along the floor, bounces off the first wall at the point *X* and hits the second wall at the point *Y*. The point *Y* is 7.5 m from the corner *C*.

The coefficient of restitution between the ball and the first wall is $\frac{3}{4}$.

Modelling the ball as a particle, find the distance *CX*.

(9)

3. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively.*]

A coastguard patrol boat C is moving with constant velocity $(8\mathbf{i} + u\mathbf{j})$ km h^{-1}. Another ship S is moving with constant velocity $(12\mathbf{i} + 16\mathbf{j})$ km h^{-1}.

(*a*) Find, in terms of u, the velocity of C relative to S.

(2)

At noon, S is 10 km due west of C.

If C is to intercept S,

(*b*) (i) find the value of u.

(ii) Using this value of u, find the time at which C would intercept S.

(4)

If instead, at noon, C is moving with velocity $(8\mathbf{i} + 8\mathbf{j})$ km h^{-1} and continues at this constant velocity,

(*c*) find the distance of closest approach of C to S.

(5)

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4. A hiker walking due east at a steady speed of 5 km h^{-1} notices that the wind appears to come from a direction with bearing 050. At the same time, another hiker moving on a bearing of 320, and also walking at 5 km h^{-1}, notices that the wind appears to come from due north.

Find

(*a*) the direction from which the wind is blowing,

(3)

(*b*) the wind speed.

(4)

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5. A particle Q of mass 6 kg is moving along the x-axis. At time t seconds the displacement of Q from the origin O is x metres and the speed of Q is v m s^{-1}. The particle moves under the action of a retarding force of magnitude $(a + bv^2)$ N, where a and b are positive constants. At time $t = 0$, Q is at O and moving with speed U m s^{-1} in the positive x-direction. The particle Q comes to instantaneous rest at the point X.

(*a*) Show that the distance OX is

$$\frac{3}{b}\ln\left(1+\frac{bU^2}{a}\right) \text{ m.}$$

(6)

Given that $a = 12$ and $b = 3$,

(*b*) find, in terms of U, the time taken to move from O to X.

(5)

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6. A particle P of mass 4 kg moves along a horizontal straight line under the action of a force directed towards a fixed point O on the line. At time t seconds, P is x metres from O and the force towards O has magnitude $9x$ newtons. The particle P is also subject to air resistance, which has magnitude $12v$ newtons when P is moving with speed v m s–1.

(*a*) Show that the equation of motion of P is

$$4\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 12\frac{\mathrm{d}x}{\mathrm{d}t} + 9x = 0.$$

(4)

It is given that the solution of this differential equation is of the form $x = \mathrm{e}^{-\lambda t}(At + B)$.

When $t = 0$ the particle is released from rest at the point R, where $OR = 4$ m.

Find,

(*b*) the values of the constants λ, A and B,

(4)

(*c*) the greatest speed of P in the subsequent motion.

(5)

7.

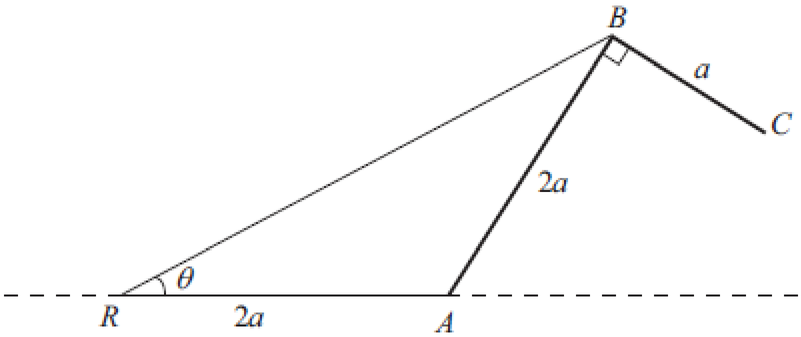


Figure 3

Figure 3 shows a framework ABC, consisting of two uniform rods rigidly joined together at B so that $\angle ABC = 90°$. The rod AB has length $2a$ and mass $4m$, and the rod BC has length a and mass $2m$. The framework is smoothly hinged at A to a fixed point, so that the framework can rotate in a fixed vertical plane. One end of a light elastic string, of natural length $2a$ and modulus of elasticity $3mg$, is attached to A. The string passes through a small smooth ring R fixed at a distance $2a$ from A, on the same horizontal level as A and in the same vertical plane as the framework. The other end of the string is attached to B.

The angle ARB is θ, where $0 < \theta < \frac{\pi}{2}$.

(*a*) Show that the potential energy V of the system is given by

$$V = 8amg \sin 2\theta + 5amg \cos 2\theta + \text{constant}.$$

(7)

(*b*) Find the value of θ for which the system is in equilibrium.

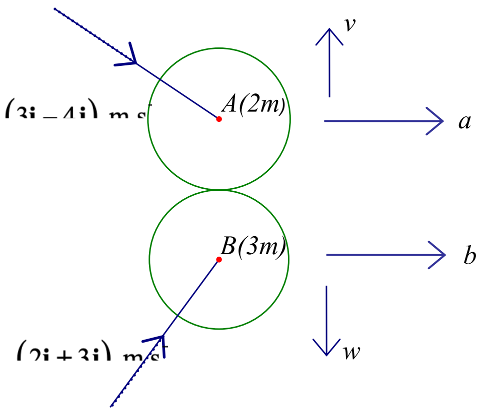
(4)

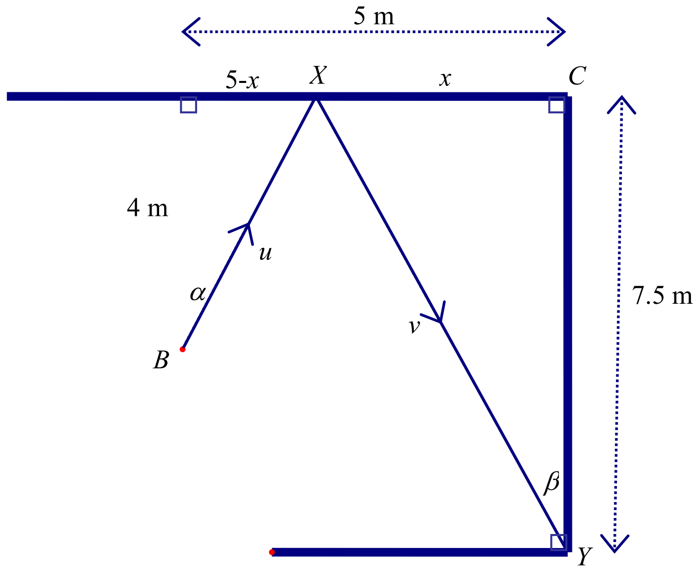
(*c*) Determine the stability of this position of equilibrium.

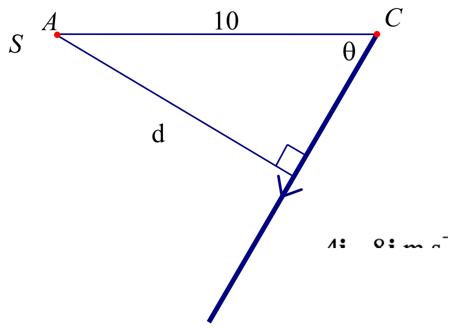
(3)

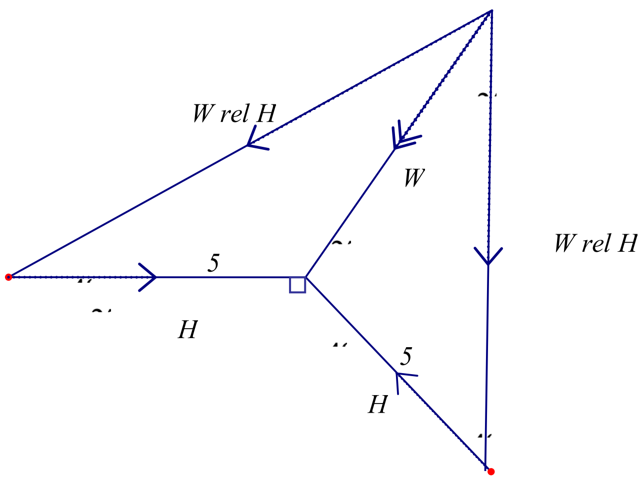
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.		
	$\leftrightarrow a = 3$ & $b = 2$	B1
	$\updownarrow$ Conservation of linear momentum: $-4\times 2+3\times 3 = 2v-3w(=1)$	M1A1
	Restitution: $v+w=\text{e}\times 7\ (=3)$	M1A1
	Solve the simultaneous equations	DM1
	giving $v=2$ and $w=1$	A1
	KE lost $=\frac{1}{2}\times 2m\times((16+9)-(4-9))+\frac{1}{2}\times 3m\times((9+4)-(1-4))$	M1A1
	$=24m\ (\text{J})$	A1
		10

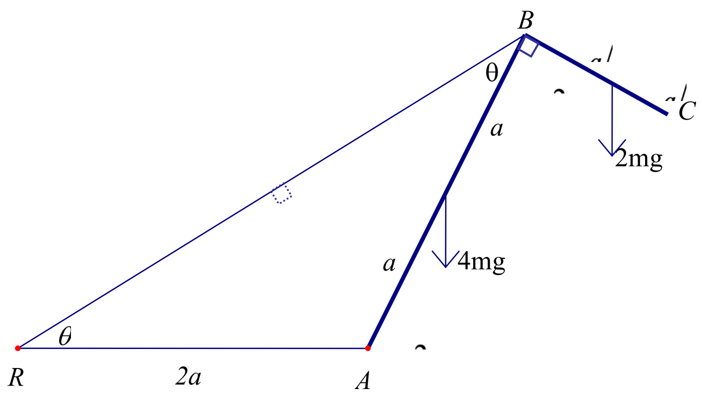
Question Number	Scheme	Marks
2.		
	At X: ↔ $u\sin\alpha = v\sin\beta$	M1A1
	↕ $v\cos\beta = eu\cos\alpha$	M1A1
	$4v\cos\beta = 3u\cos\alpha$	
	Eliminate u & v by dividing: $\frac{\tan\alpha}{3} = \frac{\tan\beta}{4}$	M1
	Substitute for the trig ratios: $\frac{5-x}{3\times 4} = \frac{x}{4\times 7.5}$	DM1A1
	Solve for x: $37.5 - 7.5x = 3x$	DM1
	$x = 3.57\ (\text{m})$ or better, $\frac{25}{7}$	A1
		9

Question Number	Scheme	Marks
3. **(a)**	Velocity of C relative to S $= (8\mathbf{i}+u\mathbf{j})-(12\mathbf{i}+16\mathbf{j})$ $= (-4\mathbf{i}+(u-16)\mathbf{j})(\text{m s}^{-1})$	M1 A1 (2)
(b) (i)	C intercepts S $\Rightarrow$ relative velocity is parallel to **i**. $\Rightarrow u-16=0,\ u=16$	M1A1 (2)
(ii)	10 km at 4 km h^{-1} takes 2.5 hours, so 2.30pm	M1A1 (2)
(c)	$u=8$, relative velocity $=-4\mathbf{i}-8\mathbf{j}$.  Correct distance identified Using velocity: $\tan\theta=\frac{8}{4}=2\Rightarrow\sin\theta=\frac{2}{\sqrt{5}}$ Using distance: $\sin\theta=\frac{d}{10}=\frac{2}{\sqrt{5}}$, $d=\frac{20}{\sqrt{5}}=4\sqrt{5}=8.9\ (\text{km})$	B1 B1 M1A1 A1 (5) **11**

Question Number	Scheme	Marks
4. (a)		
	2 vector triangles with a common side …..correct and drawn on a single diagram Wind is from bearing 025°, (N 25° E)	M1 A1 A1 (3)
(b)	$\frac{5}{\sin 25^\circ} = \frac{W}{\sin 40^\circ}$ (ft on their 25°) $W = \frac{5 \times \sin 40^\circ}{\sin 25^\circ} = 7.6 \left(\text{km h}^{-1}\right)$	M1A1ft M1A1 (4) **7**

Question Number	Scheme	Marks
5. **(a)**	Need an equation linking speed and displacement, so	
	$mv\dfrac{dv}{dx} = -(a + bv^2)$	M1A1
	Separating the variables: $\int \dfrac{6v}{a+bv^2}dv = \int -1dx$	M1
	Integrating : $\dfrac{3}{b}\ln(a+bv^2) = -x + (C)$	A1
	$X = \dfrac{3}{b}\left[\ln(a+bU^2) - \ln(a)\right] = \dfrac{3}{b}\ln\left[1 + \dfrac{bU^2}{a}\right]$ **	M1A1
	as required	(6)
(b)	Equation connecting v and t: $6\dfrac{dv}{dt} = -(12 + 3v^2)$	M1
	Separate the variables: $\int \dfrac{-6}{12+3v^2}dv = \int 1dt$	M1, A1
	$\int_U^0 \dfrac{-2}{4+v^2}\mathrm{d}v = \int_0^U \dfrac{2}{4+v^2}\mathrm{d}v = T$	M1
	$T = \dfrac{2}{2}\tan^{-1}\dfrac{U}{2} = \tan^{-1}\dfrac{U}{2}\,(\mathrm{s})$	A1
		(5) **11**

Question Number	Scheme	Marks
6. **(a)**	Using F = ma: $4\dfrac{d^2x}{dt^2} = -9x - 12v$ $= -9x - 12\dfrac{dx}{dt}$ Hence $4\dfrac{d^2x}{dt^2} + 12\dfrac{dx}{dt} + 9x = 0$ **	M1A1 M1 A1 (4)
(b)	Auxiliary eqn : $4m^2 + 12m + 9 = 0$, $(2m+3)^2 = 0, m = -3/2,\ \lambda = 3/2$ $t = 0, x = 4 \Rightarrow B = 4$ $t = 0,\ \dot{x} = e^{-\lambda t}\left(-\lambda\left(At + B\right) + A\right) = 0 \Rightarrow -6 + A = 0,\ A = 6$	B1 B1 B1 B1 (4)
(c)	$\dot{x} = e^{-\frac{3}{2}t}(-\frac{3}{2}(6t+4)+6) = -9te^{-\frac{3}{2}t}$ $\ddot{x} = e^{-\frac{3}{2}t}(-9-(-9t)\times\frac{3}{2})$, so acceleration = 0 when $t = 2/3$ at which time, $v = -6e^{-1}$, so max speed $= 6/e \approx 2.21$ m s^{-1} (3sf)	M1A1 M1 A1, A1 (5) **13**

Question Number	Scheme	Marks
7. **(a)**		
	$\text{BR} = 2\times 2a\cos\theta = 4a\cos\theta$	B1
	$\text{EPE} = 3mg\dfrac{(4a\cos\theta)^2}{2\times 2a}$	M1
	$= 12mga\cos^2\theta = 6mga + 6mga\cos 2\theta$	A1
	GPE: taking AR as the level of zero GPE,	
	GPE = GPE of AB + GPE of BC	M1+M1
	$= 4mg\times a\sin 2\theta + 2mg\left(2a\sin 2\theta - a/2\cos 2\theta\right)$	A1
	$= 8mga\sin 2\theta - mga\cos 2\theta$	
	$\Rightarrow$ Total $V = 8mga\sin 2\theta + 5mga\cos 2\theta + \text{constant}$, as required. **	A1
		(7)
(b)	$\dfrac{dV}{d\theta} = 16mga\cos 2\theta - 10mga\sin 2\theta$	M1 A1
	$\dfrac{dV}{d\theta} = 0 \Rightarrow 10\sin 2\theta = 16\cos 2\theta$	M1
	$\Rightarrow \tan 2\theta = \dfrac{8}{5} \Rightarrow \theta = 0.51$ radians (29.0°)	A1
		(4)
	Or: $8mga\sin 2\theta + 5mga\cos 2\theta = \sqrt{89}mga\cos(2\theta-\alpha),\ \tan\alpha = \dfrac{8}{5}$	M1A1
	t. pts when $2\theta - \alpha = n\pi \Rightarrow \theta = 0.51$ rads.	M1A1
(c)	$\dfrac{d^2V}{d\theta^2} = -32mga\sin 2\theta - 20mga\cos 2\theta$	M1
	$\theta = 0.51 \Rightarrow \dfrac{d^2V}{d\theta^2} < 0$, equilibrium is unstable. cso	M1A1
		(3) **14**
	Or: $2\theta - \alpha = 0 \Rightarrow \cos(2\theta - \alpha) = 1$	
	Max value $\Rightarrow$ equilibrium is unstable	